RESONANCES OF PIVOTED BLADES

MOORE FANS AND VARIABLE SPEED MOTORS

Variable speed drives are seeing ever increasing usage as an efficient way to control process temperatures by varying the airflow of an air cooler or cooling tower. Care must be taken, however, to ensure that neither fan nor drive resonances occur in the operating speed range. API Standard 661 *Air-Cooled Heat Exchangers for General Refinery Service* requires that the natural, or resonant, frequencies of the fan or fan components shall not be within 20% of the blade pass frequency. In addition to this specification, engineering design considerations dictate that the fan or its components also should not have resonances within 20% of the fan rotation speed.

This paper discusses the implications of this specification and sets out in detail the method of calculating blade resonant frequencies. The result of these calculations show that Moore fans, because of their "floating" blade design, can never produce resonances within the prohibited range.

If a fan operating on a variable speed drive has one or more resonances in the operating speed range, these speeds must be "locked out" of the controller, resulting in the need for a more sophisticated controller as well as defeating the goal of smooth, continuous process control for which variable speed drives are so useful. Figure 1 shows the effect of using a fan with a resonant frequency at 75% of the design RPM. The shaded area represents the prohibited range of fan speeds based on API 661.

Although installing such a fan on a single-speed motor would pose no problem, the difficulty arises when process conditions require running the fan within the "prohibited range" Because speeds between 60% (80% x 75%) and 90% (120% x 75%) are prohibited, airflow cannot be controlled in the range from 60% to 90% of design flow, and fan total pressure cannot be controlled in the range from 36% to 81% of design pressure. This is certainly unacceptable from a process control standpoint!

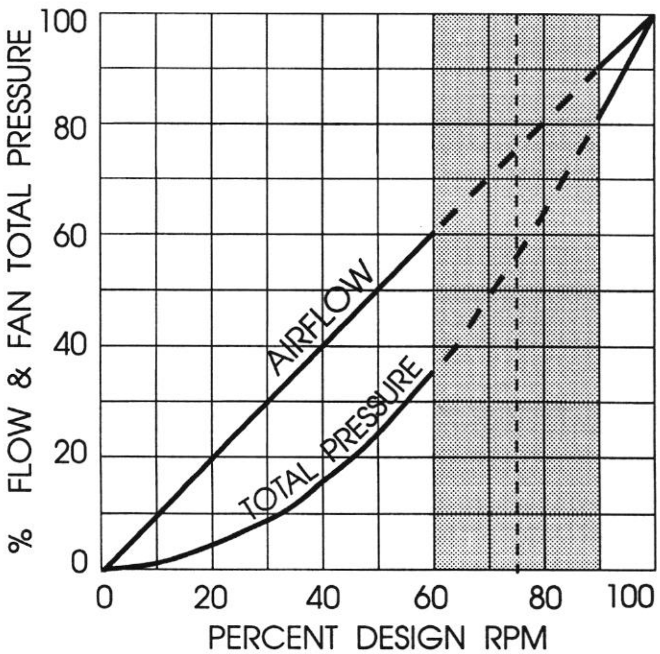

Figure 1

The vibration mode that typically gives rise to these resonances is the first vibration mode of the fan blade. In the first mode, the root of the blade is stationary, with the maximum axial displacement at the tip of the blade, as shown for a conventional fan in Figure 2. This vibration mode, typically excited by crosswinds and flow obstructions, has the lowest frequency of all vibration modes, and hence is the mode which manifests itself most often in the speed range of interest.

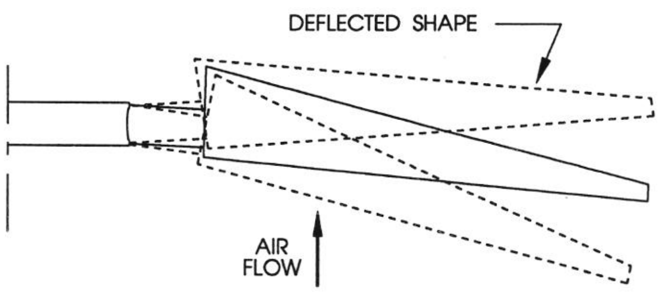

Figure 2

Moore fans, with the blade attached to the hub by a resilient mount or pivot, have a unique feature which prevents first mode resonances ever being within 20% of either the fan rotation speed or the blade pass frequency. This unique feature is purely a result of the blade/pivot dynamics which are analyzed here.

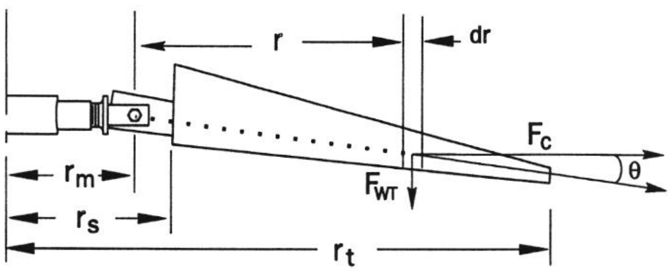

Figure 3

Figure 3 depicts a hub/blade/pivot arrangement typical of a Moore fan. The fan is rotating with an angular speed Ω. The resilient mount (pivot), with an angular spring constant K_m, is located at a radius r_m from the fan centerline. The linearly tapered blade has a root or series radius r_s, a tip radius r_t, and a mass per unit length of ρ. Each element of the blade dr at radius r from the mount has an inertia about the mount axis of:

$$I_r = \rho r^2 \, dr \tag{1}$$

which, when integrated over the entire blade yields an inertia of:

$$I = \int_{r_s}^{r_t} \rho r^2 \, dr \tag{2}$$

Also, each element is subject to a centrifugal force:

$$F_c = \rho \Omega^2 (r + r_m) \, dr \tag{3}$$

The restoring torque about the mount due to centrifugal force for each blade element is:

$$T = F_c \, r \sin\theta \qquad (4)$$

Since the angle θ is typically small, sinθ can be approximated by θ in radians. Expressing this torque as an angular spring constant (torque/angular deflection) and integrating over the entire blade yields a centrifugal spring constant:

$$K_c = \int_{r_s}^{r_t} F_c \, r \, dr \qquad (5)$$

or, from (3) and (5)

$$K_c = \int_{r_s}^{r_t} \rho\Omega^2 r^2 \, dr + \int_{r_s}^{r_t} \rho\Omega^2 r_m \, r \, dr \qquad (6)$$

Because they affect only the static droop angle θ of the blade and do not affect its dynamic response, the gravitational and airload forces can be neglected for this analysis.

Because of the taper of the blade, the mass of the blade per unit length, ρ, varies linearly from the root to the tip of the blade, or expressing this as an equation where a and b are geometric properties of the blade:

$$\rho = a - br \qquad (7)$$

Substituting this expression for ρ into (2) and (6) yields

$$K_c = \Omega^2 \left[\int_{r_s}^{r_t} (ar^2 - br^3) \, dr + \int_{r_s}^{r_t} (ar - br^2) \, r_m \, dr \right] \qquad (8)$$

$$I = \int_{r_s}^{r_t} (ar^2 - br^3) \, dr \qquad (9)$$

The natural frequency of a typical second order (inertia-spring) rotational mechanical system is simply:

$$\omega_n^2 = K / I \qquad (10)$$

or for a system with two springs acting in parallel:

$$\omega_n^2 = \omega_{n1}^2 + \omega_{n2}^2 \qquad (11)$$

where K is the total angular spring constant and I is the moment of inertia about the rotation axis. In this analysis, ω_{n1}^2 is the blade natural frequency due to the centrifugal spring effect and ω_n^2 is the natural frequency due to the resilient mount stiffness, or:

$$\omega_{n1} = \sqrt{\frac{\Omega^2 \left[\int_{r_s}^{r_t} (ar^2 - br^3) \, dr + \int_{r_s}^{r_t} (ar - br^2) \, r_m \, dr \right]}{\int_{r_s}^{r_t} (ar^2 - br^3) \, dr}} \qquad (12)$$

$$\omega_{n2} = \sqrt{\frac{K_m}{\int_{r_s}^{r_t} (ar^2 - br^3) \, dr}} \qquad (13)$$

Typically, ω_{n2}, the resonant frequency due to the resilient mount alone, is much smaller than ω_{n1} and can be neglected here to simplify the analysis. By substituting A for the rightmost integral in equation (12), and replacing the integral representing the inertia with I, equation (12) becomes:

$$\boxed{\omega_{n1} = \Omega \sqrt{1 + \frac{A}{I}}} \qquad (14)$$

It is this equation that best illustrates the key advantage of Moore fans when used with variable speed drives. It illustrates the very important point:

The blade's first mode natural frequency is never equal to the rotation speed, it is always higher. Because the expression under the radical in (14) is always greater than unity, the blade natural frequency will always be greater than the rotation speed of the fan, Ω. As process conditions change, and the fan speed changes, the natural frequency of the blade will vary along with the rotation speed, remaining always above it. This effect is purely a product of the pivot/blade dynamics. Because of this, Moore fans will never encounter a first mode resonance at the rotation speed of the fan.

To complete the analysis, we can consider the effect of the resilient mount acting in conjunction with the centrifugal spring effect. It can be seen from equation (11) that the resilient mount serves only to further increase the blade's resonant frequency.

This analysis confirms that Moore fans will never have a first mode resonance at the fan rotation speed, but the specification in API 661 concerns resonant frequencies which might occur near the blade pass frequency. The blade pass frequency Ω_{bp} is merely:

$$\Omega_{bp} = \Omega \times N$$

where N is the number of blades per fan.

The relationship between the fan series, fan diameter, blade natural frequency, fan rotation speed, and the blade pass frequency is illustrated graphically in Figure 4. The top family of curves represents the ratio:

$$\frac{\text{Blade natural frequency}}{\text{Fan rotation speed}}$$

while the bottom family of curves depict:

$$\frac{\text{Blade natural frequency}}{\text{Blade pass frequency}}$$

for varying fan series and diameters. These curves include the effect of the resilient mount stiffness previously neglected. The cross-hatched area represents the "prohibited range", or those frequencies that are within 20% of the blade natural frequency. The lower family of curves were calculated by assuming a fan with the minimum number of blades allowed for a given series. This assumption results in values closest to the "prohibited range". Adding more blades to a fan of a given series and diameter would only move the curves farther down. It can be seen that regardless of fan series or diameter, no fan will have a first mode resonance that is within 20% of either the fan rotation speed or the blade pass frequency.

The foregoing analysis has centered on the first mode resonance only. One could reasonably ask the question about the effect of higher vibration modes. Because of the extremely stiff, lightweight structure of Moore fan blades, the higher modes of vibration are well away from either blade pass or fan rotation frequencies. Moore fans have never had restricted speed ranges. This has been demonstrated by factory testing and borne out by years of use on tens of thousands of fans worldwide.

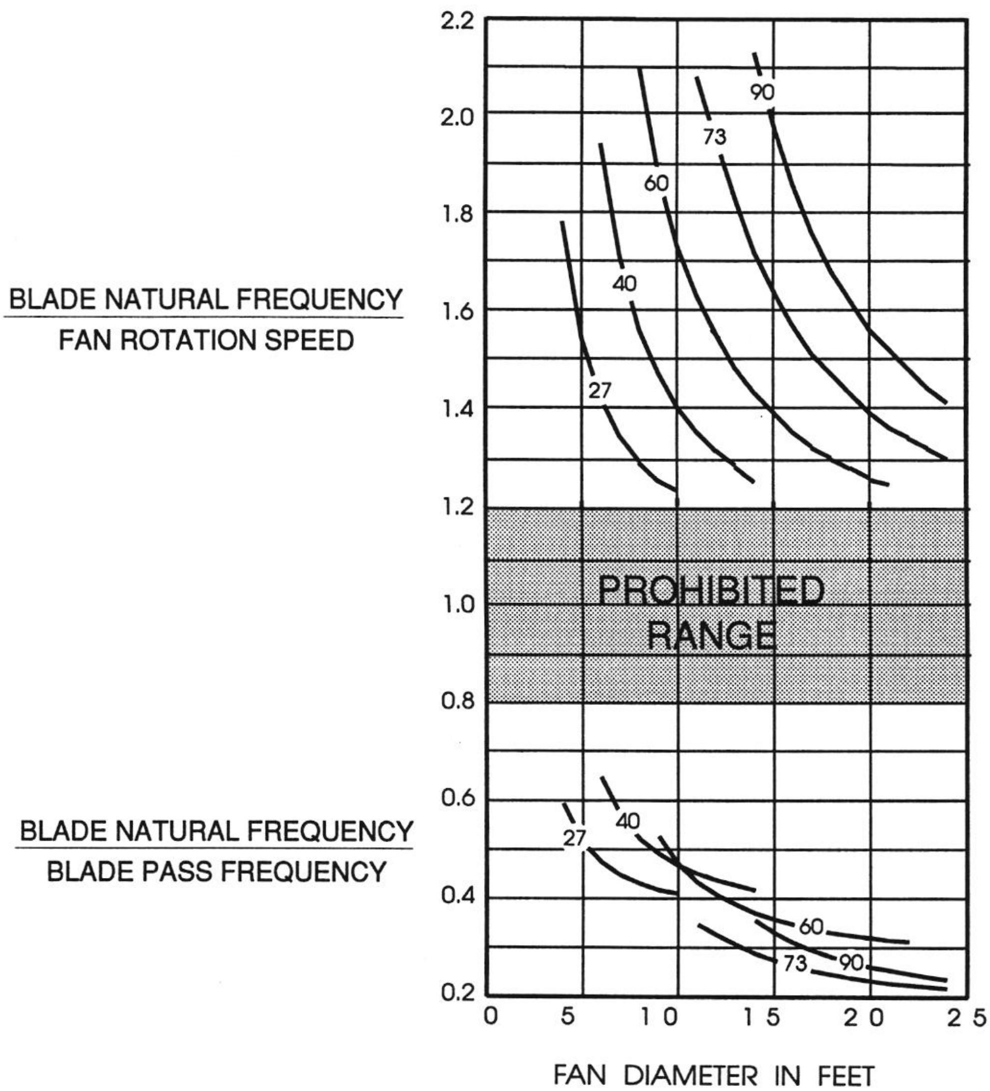

Figure 4